

Fill in the blanks.

SCORE: ____ / 10 PTS

[a] The equation of the z - axis is $\overbrace{x=y=0}^{(3)}$ and the equation of the yz - plane is $\overbrace{x=0}^{(2)}$.

[b] If $\vec{w} \cdot \vec{w} = 12$, then $\vec{w} \times \vec{w} = \underbrace{0}_{(2)}$ and $\|\vec{w}\| = \overbrace{2\sqrt{3}}^{(3)}$.
 $\|\vec{w}\|^2 = 12$

Let Q be the point $(-1, 4, -4)$, R be the point $(3, -1, 3)$,
and P be the point such that \overrightarrow{PQ} is the vector $3\vec{j} - 3\vec{k}$.

ALL ITEMS (4) POINTS SCORE: ____ / 100 PTS
UNLESS OTHERWISE NOTED

- [a] If \vec{v} is a vector of magnitude 8, and the angle between \overrightarrow{PQ} and \vec{v} is $\frac{5\pi}{6}$ radians, find $\overrightarrow{PQ} \cdot \vec{v}$.

$$\begin{aligned}\|\overrightarrow{PQ}\| \|\vec{v}\| \cos \Theta &= (3\sqrt{2})(8) \cos \frac{5\pi}{6} \quad (6) \\ &= (3\sqrt{2})(8) \left(-\frac{\sqrt{3}}{2}\right) \\ &= -12\sqrt{6}\end{aligned}$$

$$\begin{aligned}\|\overrightarrow{PQ}\| &= \sqrt{0^2 + 3^2 + (-3)^2} \\ &= \sqrt{18} \\ &= 3\sqrt{2}\end{aligned}$$

- [b] In which octant is P ?

$$\begin{aligned}\langle -1-x, 4-y, -4-z \rangle &= \langle 0, 3, -3 \rangle \\ -1-x &= 0 & x &= -1 \\ 4-y &= 3 & \rightarrow y &= 1 \\ -4-z &= -3 & z &= -1\end{aligned}$$

$$\langle -1, 1, -1 \rangle \text{ is in } O_{2+4} = O_6$$

- [c] Find a vector of magnitude 8 in the opposite direction as \overrightarrow{PR} .

$$\begin{aligned}\overrightarrow{PR} &= \langle 3-(-1), -1-1, 3-(-1) \rangle = \langle 4, -2, 4 \rangle \\ \frac{-8}{\|\langle 4, -2, 4 \rangle\|} \langle 4, -2, 4 \rangle &= \frac{8}{\sqrt{4^2 + (-2)^2 + 4^2}} \langle 4, -2, 4 \rangle \\ (7) \quad &= \frac{8}{\sqrt{36}} \langle 4, -2, 4 \rangle \\ &= -\frac{8}{3} \langle 4, -2, 4 \rangle \\ &= \left\langle -\frac{16}{3}, \frac{8}{3}, -\frac{16}{3} \right\rangle\end{aligned}$$

- [d] If $2\vec{i} - \vec{j} - c\vec{k}$ is perpendicular to \overrightarrow{PR} , find the value of c .

$$\langle 2, -1, -c \rangle \cdot \langle 4, -2, 4 \rangle = 0$$

$$(8) \quad 8 + 2 - 4c = 0$$

$$c = \frac{5}{2}$$

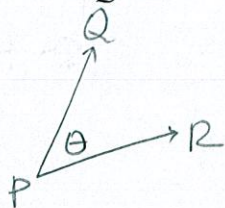
- [e] Find the volume of the parallelepiped with \overrightarrow{PQ} , \overrightarrow{PR} and $\langle 2, 1, -1 \rangle$ as adjacent edges.

$$\begin{vmatrix} 0 & 3 & -3 \\ 4 & -2 & 4 \\ 2 & 1 & -1 \end{vmatrix} \begin{vmatrix} 0 & 3 \\ 4 & -2 \\ 2 & 1 \end{vmatrix} = \underbrace{0 + 24 - 12}_{(5)} - \underbrace{(12 + 0 - 12)}_{(6)} = 12$$

- [f] If you start at point P , move 2 units to the left, 4 units down, and 6 units forward, find the co-ordinates of your ending point.

$$\underbrace{(-1+6, 1-2, -1-4)}_{(5)} = \underbrace{(5, -1, -5)}_{(6)}$$

- [g] Find $\angle QPR$.



$$\begin{aligned} \cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} &= \cos^{-1} \frac{\langle 0, 3, -3 \rangle \cdot \langle 4, -2, 4 \rangle}{(3\sqrt{2})(6)} \quad (6) \\ &= \cos^{-1} \frac{0 - 6 - 12}{18\sqrt{2}} \\ &= \cos^{-1} \frac{-18}{18\sqrt{2}} \\ &= \cos^{-1} \left[-\frac{\sqrt{2}}{2} \right] \\ &= \frac{3\pi}{4} \text{ or } 135^\circ \end{aligned}$$

Let \mathcal{P}_1 be the plane $2x - 3z = 8$,

and let \mathcal{P}_2 be the plane $3x - y - 5z = 6$.

$$\vec{n}_1 = \langle 2, 0, -3 \rangle$$

$$\vec{n}_2 = \langle 3, -1, -5 \rangle$$

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Let ℓ_1 be the line which passes through $(0, 6, -4)$ and is parallel to both \mathcal{P}_1 and \mathcal{P}_2 .

Let ℓ_2 be the line which passes through $(-4, -8, 6)$ and is parallel to ℓ_1 .

Let \mathcal{P}_3 be the plane which passes through $(2, -7, 9)$ and is perpendicular to the parallel lines ℓ_1 and ℓ_2 .

[a] Find parametric equations for line ℓ_1 .

$$\vec{d}_1 \perp \vec{n}_1 \text{ AND } \vec{n}_2$$

$$\begin{cases} x = 3t \\ y = 6 - t \\ z = -4 + 2t \end{cases}$$

SANITY CHECK:

$$\langle -3, 1, -2 \rangle \cdot \langle 2, 0, -3 \rangle = -6 + 0 + 6 = 0 \checkmark$$

$$\langle -3, 1, -2 \rangle \cdot \langle 3, -1, -5 \rangle = -9 - 1 + 10 = 0 \checkmark$$

[b] Find symmetric equations for line ℓ_2 .

$$\vec{d}_1 = -9\vec{j} - 2\vec{k} - (3\vec{i} - 10\vec{j}) = \langle -3, 1, -2 \rangle \rightarrow \text{USE } \vec{d}_1 = \langle 3, -1, 2 \rangle$$

$$\vec{d}_2 \parallel \vec{d}_1 \rightarrow \text{USE } \vec{d}_2 = \vec{d}_1$$

$$\frac{x+4}{3} = \frac{y+8}{-1} = \frac{z-6}{2}$$

$$\frac{x+4}{3} = -y-8 = \frac{z-6}{2}$$

[c] Find the standard (point-normal) equation for plane \mathcal{P}_3 .

$$\vec{n}_3 \parallel \vec{d}_1, \vec{d}_2 \rightarrow \text{USE } \vec{n}_3 = \vec{d}_1 = \vec{d}_2$$

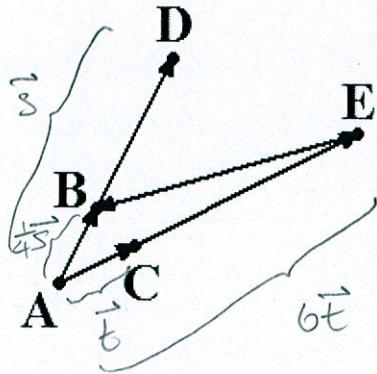
$$3(x-2) - (y+7) + 2(z-9) = 0$$

In the diagram below, ABD and ACE are both line segments.

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AE is six times the length of AC , and BD is three times the length of AB .

If $\vec{s} = \overrightarrow{AD}$ and $\vec{t} = \overrightarrow{AC}$, find an expression for \overrightarrow{EB} in terms of \vec{s} and \vec{t} .



$$6\vec{t} + \overrightarrow{EB} = \frac{1}{4}\vec{s}$$

$$\overrightarrow{EB} = \underbrace{\frac{1}{4}\vec{s}}_{\textcircled{4}} - \underbrace{6\vec{t}}_{\textcircled{2}}$$

$\textcircled{4}$